Didactic assessment over a final work of a master for mathematics teachers in service

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The aim of this paper is to present what are the criteria used by a teacher when performing the didactic analysis in its final master thesis. For that matter a case study was performed, whose object of study is the master thesis conducted by a Math teacher in service. The analysis was based on didactical suitability criteria proposed by the Onto-Semiotic Approach (OSA) to mathematical knowledge and instruction (Godino, Batanero & Font, 2007). As a result of the analysis it was possible to notice that the teacher, in addition to using all didactical suitability criteria proposed by the OSA implicitly, highlights the importance of finding a balance among the suitability criteria to achieve the learning of the didactic proposal carried out by him.

Keywords: Didactic assessment, suitability criteria, masters thesis.

Introduction

The tendency to achieve an international convergence in the planning of College programs and, particularly, those related to the Professional Master education centered on the education of teachers, has fostered a series of reforms in different countries, so that there is a model organized by a sort of refinement and evolution around professional skills. In the Brazilian context, in an attempt to train Mathematics teachers who were currently working in the area, the Professional Master Program in Mathematics in the National Network (PROFMAT) was launched in 2010 by recommendation of the *Conselho Técnico-Científico da Educação Superior da Capes*. The program is an on-site and long-distance program, offered throughout the Brazilian territory, coordinated by the *Sociedade Brasileira de Matemática* (SBM), which has a main objective to support Mathematics teachers who work in the primary education level, especially in public schools. It is important to highlight that, although PROFMAT's main objective is to foster teaching of mathematics at all levels (Brasil, 2013) it is configured as a program composed almost entirely of mathematical disciplines. In addition, at the end of the course, the students must present a final work (ETM) consisting of design a sequence of tasks, not assigning the mandatory implementation.

The work presented in this document is part of a larger research (Breda & Lima, 2016; Breda, Font & Lima, 2016; Breda, Pino-Fan & Font, 2016; Breda, Pino-Fan & Font, *in press*), in which, through the analysis of 29 Master's Thesis Works (EMTs) of PROFMAT (Breda, 2016) and was concluded that the teachers who implemented the design of their sequence of tasks performed a much more refined and balanced didactic analysis compared to the teachers who did not implement the proposal. In addition, it was evidenced that when the teachers' opinions were clearly evaluative, they were organized implicitly using some characteristics of the components of the didactical suitability criteria proposed by the Onto-Semiotic Approach (OSA) to mathematical knowledge and instruction (Godino, Batanero & Font, 2007).

So, the objective of this paper is to present a case study that analyses what are the criteria used by a teacher in his reflection process (explained in his EMT of PROFMAT), who will be addressed as Mr. Lopes, in order to improve the design and implementation of new contents related to the Riemman integral in Elementary School.

Theoretical framework

In the field of Mathematics Education there is no consensus on the notion of "quality" and, in particular, there is no consensus on the "methods for assessing and improving the teaching and learning of mathematics". There are basically two ways to address this problem, from a positivist perspective or from a consensual perspective (Font & Godino, 2011). From the first, the scientific research in the area of Mathematics Education tell us what are the causes to be modified to achieve the effects as objectives to be achieved, or at least tell us what are the conditions and restrictions that must be taken into account to achieve them. From the consensual perspective, that tells us how to guide the improvement process of mathematics instruction, which must come from the argumentative discourse of the scientific community, when it is aimed at achieving a consensus on "what can be considered as the best".

The notion of didactical suitability criteria proposed by the Onto-Semiotic Approach (OSA) to mathematical knowledge and instruction (OSA, from now onwards) (Godino, Batanero & Font, 2007) is positioned in the consensual perspective. Such notion is a partial answer to the following problematic: What criteria should be used to design a sequence of tasks to assess and develop mathematical competence of students and what changes should be made in its redesign to improve the development of this competition? Suitability criteria can first serve to guide the teaching and learning of mathematics and, second, to assess their implementation. Suitability criteria are rules of useful correction in two stages of the processes of mathematical study. A priori, the suitability criteria are the principles that guide "how things should be done". In hindsight, the criteria used to assess the study process effectively implemented. According to these authors, the didactical suitability criteria are: 1) Epistemic suitability, to evaluate if the Mathematics being taught are "good Mathematics"; 2) Cognitive suitability, to evaluate, prior to the beginning of the instruction process, if what is intended to be taught is at a reasonable distance from what the students already know, and after the process, if the knowledge acquired is any close to what was intended to teach; 3) Interactional suitability, to evaluate if the interactions contribute to clear doubts and difficulties students encounter; 4) Mediational suitability, to evaluate the adaptation of material and time-related resources used in the process of instruction; 5) Emotional suitability, to evaluate the implications (interests, motivations,…) of students during the process of instruction; 6) Ecological suitability, to evaluate the adaptation of the instruction process to the educational project of the school, the curricular guidelines, the social and professional environmental conditions (Font, Planas & Godino, 2010, p. 101).

Methodological aspects

We chose to conduct a case study (Ponte, 1994) where the didactic analysis performed by a mathematics teacher in service, as part of his master degree, is investigated. To analyse our case, we used the *indicators of didactical suitability* proposed by the OSA (Godino, Batanero & Font, 2007; Godino, 2011; Breda, Font & Lima, 2015), as theoretical model to analyse the reflections

performed by the teachers regarding ways to improve their teaching practices, related to the implementation of the didactical activities proposed as part of their EMT.

Research context

According to the guiding document of the PROFMAT, the EMT should, preferably, consist of a project with direct application to the mathematics classroom in Basic Education, thus contributing to the enrichment of the teaching of said discipline.

In this work we proceed on the assumption that the End of Master's Thesis (EMT) is a task that involves, implicitly, a didactical analysis exercise, since in their EMTs teachers must explain a didactical proposal and justify why it represents an improvement in teaching. In this sense, the reason for choosing the case of Professor Lopes is that, in addition to having applied the sequence of tasks with the students, he presents in his reflection aspects that "did not work" or that should be improved in future implementations.

Professor Lopes' didactical proposal

Professor Lopes' EMT (2014), entitled "A review of the introduction of Riemann's sums into High School Education", presents the design and the implementation of a didactic proposal for a group of third-year high school students (students aged 16 to 17) in order to intuitively introduce the integral calculus through the study of the areas of 2D geometric forms. Lopes (2014) explains that it is possible to introduce methods and notions of the integral calculus in High School Education intuitively, starting with area-calculation problems for curvilinear shapes. That is, the aim is to broaden the calculation of areas habitually studied in Elementary Education through the study of area-calculation of curvilinear shapes, using both Archimedes' and Riemann's methods.

To be specific, Lopes' EMT (2014) is organized into four chapters; in the first, the professor presents, using literature reviews, the argument: "Should integral calculus be used in Elementary School?" On the basis of this question, the professor seeks to justify- through the study of literature- the use of two methods: Archimedes' method (used to calculate the area of a circle) and Riemann's method (used to calculate the area of three curvilinear shapes: circles, eclipses and polynomial shapes with an x axis). In the second chapter, Lopes (2014) explains the didactic unit which was implemented with a group of third-year students from a state secondary/high school in Brazil. The group was formed of 41 students but at the beginning of the year, only 36 students attended the classes and participated willingly in the project. In this second chapter, the professor also explains in detail the initial self-evaluation he performed with the students and, in particular, he explains the method for evaluation previous knowledge on certain geometry topics, on mathematical software knowledge and also on the expectations of the project.

In the third chapter, professor Lopes describe the implementation he carried out. This section is a sequential report in which the author explains what happened during the implementation of the didactic sequence, placing emphasis on the set tasks, what the students learnt and the interactions made during the implementation. We are looking at a review written from the perspective of the professor but, in his very review, the professor ensures he presents evidence of the statements he makes. In the fourth and last chapter, the professor presents his reflections and conclusions on the implementation he carried out. In this way, it can be said that Lopes' proposal (2014) covers the four phases of didactic design (preliminary study, design, implementation and evaluation), which

other models of mathematics teachers' knowledge also cover, in order to answer the most fundamental question: "What knowledge should a mathematics teacher have to be able to appropriately manage their students' learning?" (Pino-Fan, Assis & Castro, 2015).

Professor Lopes' analysis on his own implementation project

When teachers have to reflect on a didactic proposal that implies a change to or an innovation in their own practices, they implicitly employ some of the *didactical suitability criteria*. Lopes' EMT (2014) has also allowed us to deduce the use of some of these criterions in the justification and reflection on the suggested proposal. In the following subsections, we show the extent to which the author considered- implicitly and explicitly- the suitability criteria put forward by OSA in attempt to defend his didactic proposal as improvement for mathematics teaching.

Epistemical suitability

Lopes (2014) justifies the 'innovative and creative' nature of his proposal by pointing out that it encourages students to perform relevant mathematical processes, in particular that of mathematical modelling. In his own words, he explains: "*In this way, the application process, divided in three stages, aims to build knowledge through the use of mathematical models. Starting with the first construction, on the basis that the topic is studied in depth and new elements arise, other models are built based on the previous ones* […]". (Lopes, 2014, p. 22)

The professor also considered that his innovative proposal allows students to perform other relevant mathematical processes such as connections, meaningful constructions, problem-solving, etc. "*In this sense, the aim is to (...) awaken the student's creativity and enthusiasm to learn geometry, to create geometrical models with the students, making connections with reality, and to provide situational problems with a geometric focus…*" (Lopes, 2014, p. 21).

It is evident in his review that some of the processes mentioned were in fact developed during the implementation of his proposal. In his thesis, the professor generally presents explicit reflections on the fact that his didactic proposal for teaching area-calculation is more representative (since it thoroughly explores the area-calculation of curvilinear figures) than the proposals that are commonly implemented at high school level.

Cognitive suitability

In Lopes' work (2014) there are comments, reflections, etc., that allow concluding that the author takes into account, in an implicit way most of the times, the indicator of cognitive suitability.

Background knowledge. The teacher carries out an initial evaluation in order to find out if the students had the necessary background knowledge for the study of the intended content. Furthermore, he makes sure that the students have such background knowledge, and specifically, he dedicates part of the time intended for the implementation, to revise the calculation of the area of triangles and quadrilaterals, and the study of trigonometric ratios. On the other hand, the learning objectives were attained by the students, "and there is confirmation that the Archimedes and Riemann methods are in the students' zone of proximal development" (Lopes, 2014, p. 19).

Curricular adaptation to individual differences. With the narration of the teacher it is not possible to conclude if he considers at some point complementary or reinforcement activities. However, when he assesses the learning related to the Riemann method, he concludes that many students will

not achieve such learning and adds: "…it would be necessary to have a more extensive study period, to be able to ask the students (…) to interpret results more thoroughly, considering that each student is unique and as such, needs a shorter or longer time to learn" (Lopes, 2014, p. 92).

Regarding the *learning* intended, the teacher states in a very clear way that he has to carry out evaluations to verify that his innovative proposal helps the students to achieve the learning objectives. Therefore, apart from the initial evaluation, the teacher carries out three formative evaluations that show the acquisition of the competences/learning implemented. With these evaluations, the teacher concludes that the learning related to the calculation of areas of quadrilaterals and triangles, and the Archimedes method was acquired, but the same cannot be said about the learning of the Riemann method, which he justifies with lack of time.

High cognitive demand. The author considers that his proposal requires a high cognitive demand from the students, since it activates relevant cognitive processes.

Interactional suitability

Teacher-Student interaction. The teacher describes a "teacher-large group interaction", through a dynamic of questions asked by the teacher and answers given by the students, which, according to him "facilitates comprehension among students" (Lopes, 2014, p. 32). He also presents some examples of how this type of interaction helps to clarify doubts that the students might have.

Interaction among students. In his narrative, the teacher also mentions that the students worked in small groups and although he did not comment if such dynamic has solved the student's semiotics conflicts, he concludes that this organization allowed some students that hardly participated in the classroom to express themselves in a larger group.

Autonomy. It is possible to conclude that there were moments in which the autonomy of students was encouraged. On the one hand, "the students had to do homework" (Lopes, 2014, p. 67); on the other hand, there were some moments in which it was possible to observe that the responsibility to study (exploration, formulation and validation) was assumed by the students.

Formative evaluation. As mentioned in the cognitive suitability section, the teacher carried out a formative evaluation that allowed a systematic observation of the cognitive process of the students.

Mediational suitability

It was possible to observe the use of *material resources* such as calculators and computers. The teacher explains that he used the *GeoGebra* software and the calculator during the teaching process. Regarding *GeoGebra*, he presents some implicit evaluative comments about the advantages of including this software of dynamic geometry in the teaching process.

Number of students, Schedule and classroom conditions. Regarding this aspect, the teacher makes several comments. In a relevant way, he explains that the number of students and the conditions of the classroom (both the physical space as well as the computer laboratory) somehow determined the use of *GeoGebra*. Thus, the software was mainly used by the teacher to illustrate and show mathematical practices (e.g., the calculation of the areas of quadrilaterals and triangles).

Regarding the *time – of group teaching and learning –,* the teacher makes comments and assessments about three indicators of this component: the adaptation of intended meanings in the

available time, the time spent in the most important and relevant contents, and the time spent in the contents that were more difficult for the students. In connection to the first indicator, the teacher states very clearly that he could not adapt the intended meanings in the time that was available. Particularly, he states that he did not have enough time to finish explaining all he had planned regarding the Riemann method. For the second indicator, the teacher states that it took him a lot of time to ensure the required background knowledge, and that, on the other hand, he did not have time to solve the initial problem that was contextualized in order to later introduce the Archimedes and Riemann methods. Finally, regarding the third indicator, it is possible to infer from the teacher's comments that it was impossible to carry out the whole study due to lack of time (e.g., there was not enough time to explain the Riemann method in depth).

Emotional suitability

In connection to this suitability indicator, no comments regarding the *interests and needs of the students* were found in Lopes' EMT (2014). No comments about *the attitudes* of the students were found neither. Regarding *emotions*, the teacher states that the implementation he carried out promotes the students' self-esteem.

Ecological suitability

According to the criteria and objectives that the teachers had to consider for the elaboration of their projects, professor Lopes adds that his proposal is a *didactical innovation* that adapts to the *Elementary school* curriculum and, according to his students, contributes to social and professional integration (*social and labour utility*) and that presents an intra-mathematical *connection* to higher level Mathematics (*intra and interdisciplinary connections*).

Final reflections

The analysis of the EMT of Professor Lopes shows how the indicators of *didactical suitability* proposed by OSA are -implicitly- present in his reflection processes on their own practice. An important aspect to highlight is that this EMT clearly demonstrates the issue of finding a balance between each of the suitability criteria. On one hand, the author plans an innovation with high epistemic suitability and he demonstrates in his review that he also made a substantial effort to achieve high cognitive suitability. On the other hand, however, he also demonstrates that he was obliged to neglect part of the content he had planned; in particular he could not solve the initial problem which was the very motive of his didactic proposal and the learning was not complete (in particular, the Riemann method) due to the fact that the suitability of means was not adequate; to be precise, there was not enough time.

We could say that, in terms of the suitability criteria, Lopes concludes that if in future implementations, cognitive and epistemic suitability are not to be neglected, and then it would be necessary to allow more time. One aspect, which is difficult to explain, is the reason why the criteria of didactic suitability function as implicit patterns in the Lopes' discourse, when he has to evaluate instruction processes without specific training on the use of this analysis tool. One possible explanation is that the training that he has received in the PROFMAT allows him to do, implicitly, this type of analysis. However, Caldatto's research (2015) and Caldatto Pavanello and Fiorentini (2016), leads us to believe that the characteristics of the PROFMAT do not encourage this kind of

reflection. Now, other answer to this very question is related to the origins of this construct. In the OSA, the didactic suitability criteria, its components and characteristics were constructed on the basis that they should be constructs which rely on a certain amount of consensus within the Mathematics Education community, albeit the local one. Therefore, one of the plausible explanations that the suitability criteria can be considered as teachers' reflections patterns is related to the extensive consensus that they themselves generate amongst persons involved in Mathematics Education. Therefore, another possible explanation, for Lopes case, could be based on his previous training and his experience, which would lead him to participate in this consensus.

The analysis of the EMT of Professor Lopes shows that, when the teachers' opinions were clearly evaluative, they were organized implicitly using some characteristics of the components of the didactical suitability criteria proposed by the OSA. This result has been evidenced in other investigations (Breda, 2016; Seckel, 2016; Breda, Pino-Fan, Font, *in press*) in which it is also suggested that the suitability indicators can be taught as powerful methodological tools to organize the teacher reflection –as it has already been done in different processes of teacher training in Spain, Ecuador, Chile and Argentina (Giménez, et al., 2012; Valls & Vanegas, 2015; Posadas, 2015; Pochulu, Font & Rodriguez, 2016; Seckel, 2016) –, that aim at the fostering of the "meta" dimension of didactical-mathematical knowledge (DMK) of Mathematics teachers (Pino-Fan, Assis & Castro, 2015; Pino-Fan, Godino & Font, 2016).

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References

Brasil. (2013). Uma análise quali-quantitativa de perfis de candidatos ao Mestrado Profissional em Matemática em Rede Nacional (PROFMAT). Sociedade Brasileira de Matemática (SBM).

Breda, A. (2016). *Melhorias no ensino de matemática na concepção de professores que realizam o mestrado PROFMAT no Rio Grande do Sul: uma análise dos trabalhos de conclusão de curso*. Tese de doutorado, Pontifícia Universidade Católica do Rio Grande do Sul, Porto Alegre, Brasil.

Breda, A., Font, V., & Lima, V. (2015). A noção de idoneidade didática e seu uso na formação de professores de matemática. *Jornal Internacional de Estudos em Educação Matemática, 8*(1), 4-41.

Breda, A., Font, V., & Lima, V. M. (2016). Análise das Propostas de Inovação nos Trabalhos de Conclusão de Curso de um Programa de Mestrado Profissional em Matemática. *Avances de Investigación en Educación Matemática, 10*(2), 53-72.

Breda, A., & Lima, V. M. R. (2016). Estudio de caso sobre el análisis didáctico realizado en un trabajo final de un máster para profesores de matemáticas en servicio. *REDIMAT - Journal of Research in Mathematics Education*, *5*(1), 74-103. doi: http://dx.doi.org/10.17583/redimat.2016.1955

Breda, A., Pino-Fan, L., & Font, V. (2016). Establishing criteria for teachers' reflection on their own practices. In Csíkos, C., Rausch, A., & Szitányi, J. (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 283). Szeged, Hungary: PME.

Breda, A., Pino-Fan, L., & Font, V. (*In press*). Meta didactic-mathematical knowledge of teachers: criteria for the reflection and assessment on teaching practice. *Eurasia Journal of Mathematics, Science & Technology Education*.

Caldatto, M. (2015). *O PROFMAT e a formação do professor de matemática: uma análise curricular a partir de uma perspectiva processual e descentralizadora*. Tese de doutorado, Universidade Estadual de Campinas, Campinas, Brasil.

Caldatto, M., Pavanello, R., & Fiorentini, D. (2016). O PROFMAT e a Formação do Professor de Matemática: uma análise curricular a partir de uma perspectiva processual e descentralizadora. *Bolema, 30*(56), 906-925.

Valls, S. F., & Muṇoz, Y. M. V. (2015). Uso de criterios de calidad en la reflexión sobre la práctica de los futuros profesores de secundaria de matemáticas. *Procedia-Social and Behavioral Sciences*, *196*, 219-225.

Font, V. & Godino, J. D. (2011), Inicio a la investigación en la enseñanza de las matemáticas en secundaria y bachillerato, en J. M. Goñi (ed.), *MATEMÁTICAS: Investigación, innovación y buenas prácticas* (9-55). Barcelona, España, Graó.

Giménez, J., Vanegas, Y., Font, V., Ferreres, S. (2012). El papel del trabajo final de Máster en la formación del profesorado de Matemáticas. *UNO. Revista de Didáctica de las Matemáticas*, *61*, 76-86.

Godino, J. D. (2011). Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. *XIII Conferência Interamericana de Educação Matemática (CIAEM-IACME).* Recife, Brasil.

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM. The International Journal on Mathematics Education, 39*(1), 127 – 135. doi: 10.1007/s11858-006-0004-1

Lopes, A. (2014). *Um relato sobre à introdução às somas de Riemann na Educação Básica*. Dissertação de mestrado, Mestrado Profissional em Matemática em Rede Nacional PROFMAT, Universidade Federal de Santa Maria, Santa Maria, Brasil.

Pino-Fan, L., Assis, A., & Castro, W. F. (2015). Towards a methodology for the characterization of teachers' didactic-mathematical knowledge. *EURASIA Journal of Mathematics, Science & Technology Education*, *11*(6), 1429-1456. doi: 10.12973/eurasia.2015.1403a.

Pino-Fan, L., Godino, J. D., & Font, V. (2016). Assessing key epistemic features of didactic-mathematical knowledge of prospective teachers: the case of the derivative. *Journal of Mathematics Teacher Education*. doi: 10.1007/s10857-016-9349-8

Pochulu, M., Font, V. & Rodríguez, M. (2016). Desarrollo de la competencia en análisis didáctico de formadores de futuros profesores de matemática a través del diseño de tareas. *Revista Latinoamericana de Investigación en Matemática Educativa-RELIME*, *19*(1), 71-98.

Posadas, P. (2013). *Evaluación de la idoneidad didáctica de una experiencia de enseñanza sobre ecuaciones de segundo grado en 3º de educación secundaria obligatoria*. Tesis de magíster, Universidad de Granada, Granada, España.

Ponte, J. P. (1994). O estudo de caso na investigação em educação matemática. *Quadrante*, *3*(1), 3-18.

Seckel, M. J. (2016). *Competencia en análisis didáctico en la formación inicial de profesores de educación general básica con mención en matemática.* Tesis de doctorado, Universitat de Barcelona, Barcelona, España.