MATHEMATICS TEACHERS' KNOWLEDGE AND COMPETENCES MODEL BASED ON THE ONTO-SEMIOTIC APPROACH

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This paper aims at developing a model that intends to articulate diverse categories of mathematics teachers' knowledge and competences that are necessary for the appropriate teaching of mathematics, based on the theoretical notions of the Onto-Semiotic Approach to mathematical knowledge and instruction (OSA) and its many contributions to the fields of teacher training.

INTRODUCTION

The study of the didactic and mathematical knowledge and competences that a teacher should have to appropriately manage the students' learning process is a matter that has been largely researched, thus, generating several model designs that aim at characterizing such teachers' knowledge and competences (e.g., Shulman, 1987; Rowland, Huckstep & Thwaites, 2005; Hill, Ball & Schilling, 2008; Schoenfeld & Kilpatrick, 2008). Based on the theoretical notions of the Onto-Semiotic Approach (OSA) to mathematical knowledge and instruction (Godino, Batanero & Font, 2007) and its many contributions to the field of teacher training, this work develops a model (called Didactic-Mathematical Knowledge and Competences model or DMKC) that intends to articulate the diverse categories of teachers' knowledge and competences that are necessary for the appropriate teaching of Mathematics, and at the same time, refines the DMK model presented in Pino-Fan, Assis & Castro (2015).

DIDACTIC-MATHEMATICAL KNOWLEDGE AND COMPETENCES MODEL

A theoretical model of teachers' mathematical knowledge (Pino-Fan, Assis & Castro, 2015; Pino-Fan, Godino & Font, 2016) within the framework of the Onto-Semiotic Approach (OSA) to mathematical knowledge and instruction (Godino, Batanero & Font, 2007) has already been designed and is known as the DMK model. As stated by its authors, one of the perspectives of development of this model is the fitting of the notion of teachers' knowledge and teachers' competences. On the other hand, also within the framework of OSA, there have been other studies regarding Mathematics teachers' competences (Rubio, 2012; Giménez, Font, & Vanegas, 2013; Seckel, 2016; Pochulu, Font & Rodríguez, 2016), which have also exposed the need of having a model of teachers' knowledge to evaluate and develop their competences. These two research agendas have come together, thus generating the mathematics teachers' *didactic-mathematical knowledge and competences model* (DMKC model) (Breda, Pino-Fan & Font, *in press*).

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The notion of competence

Mathematics teachers are expected to be able to address basic didactic problems related to the teaching of this subject through the use of theoretical and methodological tools, giving way to a series of specific competences. Thus, the two first key questions that arise, in order to be able to develop the DMKC model, are: What is understood by the notion of competence? What are the key competences that Mathematics teachers should have? According to Weinert (2001), competency-based approaches can be classified into three large groups: a) Cognitive approach; b) Motivational approach; and c) integral approach or action competence. According to this, the conceptualization of competence that we use in this model comes from the action competence perspective, considering it as a combination of knowledge, dispositions, etc., that allows an effective performance, within typical contexts of the profession, of the actions aforementioned in its formulations. In an Aristotelian way, it is about a potentiality that is updated in the performance of effective actions (competences).

This formulation of the term "competence" has to be developed in order to be operational, and for that purpose it is necessary to characterize competence (definition, levels of development and descriptors) that allows its development and evaluation. According to Seckel (2016), we consider that the starting point for the development and evaluation of a professional competence has to be a task that generates the perception of a professional problem that needs to be solved, and for this purpose, the prospective teacher or in-service teacher has to mobilize skills, knowledge and attitudes in order to develop a practice (or action) that intends to solve the problem. Furthermore, we can expect such practice to be performed with more or less success (achievement) and, at the same time, it can be considered as evidence that the person can perform practices that are similar to the ones described by some descriptors of the competence, which is often associated to a certain level of competence.

Mathematical competence and competence in analysis and didactic intervention

Students' mathematical competences are developed through the solving of mathematical tasks and, at the same time, evaluated through the mathematical activity performed in order to solve the assigned task. In the case of evaluation, the teacher assigns a task to the student, who solves it by performing a certain mathematical activity. Then, the teacher analyses the student's mathematical activity and finds evidence of a certain level of development of one or several mathematical competences. In Rubio (2012), it is stated that, in order to evaluate their students' mathematical competences, teachers must have mathematical competence. However, it is also stated that this is not enough, since the teacher must also be competent in the analysis of mathematical activity. While the first competence is not specific to the teaching profession (it is common in several professions that use mathematics, although each profession gives it its hallmark), the second one, as a matter of fact, is.

The DMKC model considers that the two key competences of Mathematics teachers are *Mathematical competence* and *Competence in analysis and didactic intervention*, which, at its core (Breda, Pino-Fan & Font, *in press*) consists of: *Designing, applying and assessing sequences of one's own learning and others', through techniques of didactic analysis and quality criteria to establish periods of planning, implementation, assessment and outline suggestions for improvements*. In order to be able to develop this competence, the teacher needs, on the one hand, knowledge that allows to describe and explain what is happening in the process of teaching and learning (didactical dimension of the DMK model, one of the components of the DMKC model), and on the other hand, needs knowledge to assess what has already happened and outline suggestions for improvements in future implementations −meta didactic-mathematical dimension of the DMK model, one of the components of the DMKC model (Pino-Fan, Assis & Castro, 2015). In this work, we will focus mainly on the latter.

CHARACTERIZATION OF THE COMPETENCE IN ANALYSIS AND DIDACTIC INTERVENTION

This general competence is formed by different sub-competences (Breda, Pino-Fan & Font, *in press*): 1) sub-competence in analysis of the mathematical activity; 2) sub-competence in analysis and management of the interaction and its effect on students' learning; 3) sub-competence in analysis of norms and meta-norms; and 4) sub-competence in assessment of the didactical suitability of the process of instruction.

Sub-competence in analysis of mathematical activity

Rubio (2012) describes the design and implementation of a training period in the Secondary School Teachers Training Master Program of Universitat de Barcelona, in which teachers are first taught the technique for the analysis of practices, objects and processes proposed by OSA, and then, a technique for the evaluation of mathematical competences. The objective of this study was to corroborate (or not) the following hypothesis: the professional competence of teachers in the analysis of mathematical practices and mathematical objects and processes activated in such practices, is "in-depth knowledge" that allows to evaluate and develop the students' mathematical competences. Rubio (2012) concludes that after all the experiments conducted, such hypothesis can be confirmed. Furthermore, it is stated that if teachers are not competent in the analysis of mathematical practices, processes and objects, they will not be competent in the evaluation of mathematical competences. Thus, the results of Rubio's thesis (2012) point out a sub-competence of the competence in analysis and didactic intervention that mathematics teacher have to develop in order to develop and evaluate their students' competences: competence in analysis of the mathematical activity, in other words, the analysis of the mathematical practices, objects and mathematical processes activated in them.

This first sub-competence enables teachers to analyze mathematical activity. This type of analysis is important in the training of teachers and is a type of analysis that is somehow difficult for teachers and future teachers. For example, Stahnke, Schueler and Roesken-Winter (2016) carry out a revision of the empirical research conducted on mathematics teachers and conclude that teachers have difficulty analyzing the mathematical tasks (and its educational potential) assigned to their students.

As mentioned before, the lack of consensus over a paradigm that defines how should the analysis of mathematical activity be done in the field of mathematical education is a very problematic aspect. The DMKC model assumes that the theoretical tools of OSA (practice, primary and secondary objects emerging from the practices, meaning of a mathematical object in terms of practices, partial meanings, mathematical processes) allow such analysis in terms of practices, mathematical objects and processes. With these theoretical notions, when the meanings are understood pragmatically in terms of practices, one can, firstly, answer questions such as: What are the partial meanings of the mathematical objects that are intended to be taught? How are they articulated together? Later, an analysis of the primary mathematical objects and processes activated in such practices can be conducted. The identification by part of the teacher of the objects and processes involved in mathematical practices allows to comprehend the progression of the learning process, to manage the necessary processes of institutionalization and to evaluate the students' mathematical competences. Thus, it is possible to answer the questions: What are the configurations of primary mathematical objects and processes involved in the practices that constitute the diverse meanings of the intended contents (epistemic configuration)? What are the configurations of primary objects and processes used by students when solving problems (cognitive configurations)? Mathematics teachers have to know and comprehend the idea of configuration of objects and processes activated in a certain mathematical practice and be able to use it in a competent manner in the processes of teaching and learning mathematics (Pino-Fan, Godino & Font, 2016).

Sub-competence in analysis and management of the interaction and its effect on students' learning

The notion of didactic configuration has been introduced in OSA as a tool for the analysis of the interactions in instruction processes (Godino, Contreras & Font, 2006). It is about a theoretical construct to model the articulation of the performance of teachers and students regarding a specific task and content (a configuration of primary objects and processes) of teaching and learning, where knowledge arises from the interaction itself. Mathematics teachers have to be competent in the design and management of didactic configurations. It intends to answer the following question: What type of interactions between people and resources will be implemented in instructional processes and what are the consequences in the learning process? How can interactions and conflicts be managed in order to optimize learning? The teacher, therefore, should know the many types of didactic configurations (dialogic, etc.) that can be implemented and their effect on students' learning, and also, how to design and manage these types of didactic configurations in specific instruction processes.

Sub-competence in normative analysis

The different stages of the process of design and implementation are supported by and depend on a complex net of norms and meta-norms of different origin and nature (Godino, Font, Wilhelmi & Castro, 2009) that need to be explicitly recognized in order to comprehend the development of instruction processes and direct them towards optimal suitability levels. For example, when studying equations, there are rules regarding the way these should be written or the way these should be solved. Also, there are non-mathematical norms, such as the use (or not) of calculators, the method of evaluation, the way of participating in class, etc. Mathematics teachers have to become competent in the normative analysis of the processes of mathematical instruction in order to answer questions such as: what norms determine the development of instructional processes? Who, how and when are the norms established? What and how can these be changed in order to optimize mathematical learning? Etc.

Sub-competence in the assessment of the didactical suitability of instruction processes

The characterization of the competence in analysis and didactic intervention proposed above, needs tools for the description and explanation, as those described in Rubio's research study (2012), for the analysis of mathematical activity and also tools for assessment, as those presented in the research studies conducted by Seckel (2016) and Breda, Pino-Fan and Font (*in press*). These research studies show that, even when the teachers do not know the didactical suitability criteria with all their components and indicators, if they are exposed to a situation in which they have to assess a proposal of didactic innovation that could somehow affect them, then they use them in an implicit way to organize their positive or negative assessment.

For the assessment of instruction processes, OSA proposes didactic suitability as the essential tool. Once a specific topic has been selected in a certain educational context, the notion of didactic suitability (Breda, Font & Lima, 2015) helps to answer questions such as: what is the degree of didactical suitability of the teaching and learning processes implemented? What changes should be made in the design and implementation of the instruction process in order to increase its didactic suitability in future implementations?

Didactical suitability of an instruction process is defined as the degree to which such process (or a part of it) gathers certain characteristics that enables it to be assessed as suitable (optimal or ideal) to attain the adaptation between the personal meanings achieved by the students (learning) and the intended or implemented institutional meanings (teaching), taking into account the circumstances and available resources (environment). The notion of didactical suitability can be separated into six specific suitabilities: 1) Epistemic suitability that makes reference to the mathematics taught as ideally be "good mathematics". For that purpose, apart from taking the prescribed curriculum as reference, it also considers the institutional mathematics that have been transposed into the curriculum; 2) Cognitive suitability, that expresses the degree to which the intended or implemented learning is within the students' zone of potential development, and also the proximity of the attained learning to the learning intended or implemented; 3) Interactional suitability, that refers to the degree to which the modes of interaction allow to identify and solve conflicts of meaning and favor autonomy in learning; 4) Mediational suitability, the degree of availability and adaptation of the material and time resources necessary for the development of the teaching and learning processes; 5) Affective suitability or degree of implication (interest, motivation) of students in the process of study; and 6) Ecologic suitability, degree of adaptation of the process of study to the school comprehensive education plan, the curricular guidelines, the environment, etc.

For each of these criteria, there is a system of components and indicators that can be rated on a scale (of 1–3, for example). It is about a system of rubrics that allows to rate (or auto rate) in a complete or balanced way, the elements that, together, make up a process of quality instruction in the field of mathematics.

KNOWLEDGE OF MATHEMATICS TEACHERS

Professional competences have to be developed in the training of teachers. For that purpose, the teacher trainer has the capacity of analyzing the professional practices of teachers (future teachers or in-service teachers) when they solve professional tasks assigned to them in a training period, and the didactic-mathematical knowledge activated in them, in order to be able to find indicators that justify the assignation of degrees of development of the professional competence that is being evaluated. However, a problem that we have in the field of mathematics education is that there is not a single model that allows us to analyze the professional practice and there is no consensus over a paradigm for the analysis of the didactic-mathematical knowledge activated by teachers in their professional practices.

As discussed in the first section, there are several models and views worldwide regarding the knowledge that mathematics teachers should have in order to appropriately manage their students' learning. Pino-Fan, Assis and Castro (2015) propose a model for characterizing didactic-mathematical knowledge (DMK) of teachers, which considers, among other aspects, the contributions and developments of several models of mathematics teachers' knowledge, and the theoretical and methodological development of OSA. Thus, the DMK model suggests that teachers' knowledge is organized into three dimensions: 1) mathematical; 2) didactical; and 3) meta didactic-mathematical. The first dimension, mathematical, refers to the knowledge that enables teachers to solve mathematical problems or tasks that are typical of the educational level in which they will teach (common knowledge), and link the mathematical objects of such level to mathematical objects that will be studied at higher levels (extended knowledge) (Ibíd., p. 1433).

The didactical dimension of DMK proposes six subcategories of teachers' knowledge (Ibíd., p. 1434-1436): 1) epistemic facet, that refers to the specialized knowledge of mathematical dimension (use of diverse representations, arguments, procedures, partial meanings for a specific mathematical object…); 2) cognitive fact, that refers to the knowledge about cognitive aspects of students (difficulties, errors, conflicts, learning...); 3) affective facet, that refers to the knowledge of affective, emotional and attitudinal aspects of students; 4) interactional facet, knowledge of the interactions that occur in the classroom (teacher-student, student-student, student-resources...); 5) mediational facet, knowledge of the resources and means that can foster the students' learning process, and the time assigned for teaching processes; and 6) ecologic facet, knowledge of curricular, contextual, social, political, economical aspects that may have influence on the students' learning process.

The third dimension of DMK, meta didactic-mathematical dimension, refers to the knowledge needed by teachers to: reflect on their own practice, identify and analyze the set of norms and meta-norms that regulate the teaching and learning processes of mathematics, and assesses the didactic suitability in order to find potential improvements in both, design and implementation stages of such processes (Pino-Fan, Assis & Castro, 2015; Pino-Fan, Godino & Font, 2016).

The three dimensions described above are involved in the different phases of the design of processes of teaching and learning of specific mathematical topics: preliminary study, planning or design, implementation and assessment (Pino-Fan, Godino & Font, 2016).

FINAL CONSIDERATIONS

This work has presented a theoretical model, the mathematics teachers' Didactic-Mathematical Knowledge and Competences model (DMKC model), which is based on a series of empirical research studies that, on the one hand, have allowed its development and refinement and, on the other hand, have tested its theoretical constructs. Although the work that has been presented is basically theoretical, it is important to highlight that there have been a number of empirical research studies on the diverse components of the model, as can be seen in the section "Formación de profesores" (Teacher training) on the OSA website: http://enfoqueontosemiotico.ugr.es/. The DMKC model opens, therefore, a strong research program and development focused on the design, experimentation and evaluation of formative interventions that promote the professional development of mathematics teachers, taking into account the different categories of knowledge and didactic competences described in this work.

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References

Breda, A., Font, V., & Lima, V. M. R. (2015a). *A noção de idoneidade didática e seu uso na formação de professores de matemática*. Jornal Internacional de Estudos em Educação Matemática, 8(2), 4-41.

Breda, A., Pino-Fan, L., & Font, V. (*In press*). Meta didactic-mathematical knowledge of teachers: criteria for the reflection and assessment on teaching practice. *Eurasia Journal of Mathematics, Science & Technology Education.*

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM Mathematics Education, 39*(1), 127 – 135. doi: 10.1007/s11858-006-0004-1

Godino, J. D., Font, V., Wilhelmi, M. R. & Castro, C. (2009). Aproximación a la dimensión normativa en didáctica de las matemáticas desde un enfoque ontosemiótico. *Enseñanza de las Ciencias*, 27(1), 59–76

Hill, H. C., Ball, D. L., & Schlling, S. G. (2008). Unpacking pedagogical content knowledge of students. *Journal for Research in Mathematics Education*, 39, 372–400.

Pino-Fan, L., Assis, A., & Castro, W. F. (2015). Towards a methodology for the characterization of teachers' didactic-mathematical knowledge. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(6), 1429-1456. doi: 10.12973/eurasia.2015.1403a

Pino-Fan, L., Godino, J. D., & Font, V. (2016). Assessing key epistemic features of didactic-mathematical knowledge of prospective teachers: the case of the derivative. *Journal of Mathematics Teacher Education*. doi:10.1007/s10857-016-9349-8

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.

Rubio, N. (2012). *Competencia del profesorado en el análisis didáctico de prácticas, objetos y procesos matemáticos*. (Doctoral dissertation). Retrieved from http://www.tdx.cat/handle/10803/294031

Seckel, M. J. (2016). *Competencia en análisis didáctico en la formación inicial de profesores de educación general básica con mención en matemática*. (Doctoral dissertation). Retrieved from http://www.tdx.cat/handle/10803/385915

Schoenfeld, A., & Kilpatrick, J. (2008). Towards a theory of profiency in teaching mathematics. En D. Tirosh, & T. L. Wood (Eds.), *Tools and processes in mathematics teacher education* (pp. 321-354) Rotterdam: Sense Publishers.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Stahnke, R., Schueler, S. & Roesken-Winter, B. (2016). *ZDM Mathematics Education* 48: 1-27. doi:10.1007/s11858-016-0775-y

Weinert, F. (2001). Concept of competence: A conceptual clarification. In D. Rychen & L. Salganik (Eds.), *Definition and selection key competencies* (pp. 45–65). Gottingen: Hogrefe & Huber.